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Some I-convergent triple sequence spaces defined by a sequence of modulus function

Bimal Chandra Das
Tripura University, India
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Abstract

In this article we introduce the notion of I-convergent triple sequence spaces $c^3_{0I}(F)$, $c^3_I(F)$, $l^3_{\infty I}(F)$, $m^3_I(F)$ and $m^3_{0I}(F)$ defined by a sequence of modulii $F = (f_{pqr})$ and study some of their algebraic and topological properties like solidity, symmetricity, convergence free etc. We also prove some inclusion relation involving these sequence spaces.

Key Words : *Triple sequence, Modulus function, I- convergence, Ideal, filter*

AMS Classification : *40A05, 40A35, 40C05, 46A45.*

1. Introduction

Throughout the article a triple sequence x is denoted by (x_{pqr}) i.e. a triple infinite array of real or complex numbers x_{pqr}, $p, q, r \in N$. Throughout the article $c^3_{0I}(F)$, $c^3_I(F)$, $l^3_{\infty I}(F)$, $m^3_I(F)$ and $m^3_{0I}(F)$ denote the triple sequence spaces of I- null in Pringshiems sense , I- convergent in Pringshiems sense, I- bounded in Pringshiems sense, bounded I-convergence and bounded I-null in Pringshiems sense respectively. Throughout N, R and C denote the set of natural, real and complex numbers respectively.

Let w^3 denote the set of all triple sequences of real or complex numbers. Then the classes of triple sequences c^3_0, c^3, l^3_∞, c^{3R} and c^{3B} denote the triple sequence spaces which are convergent to zero in Pringsheim′s sense, convergent in Pringsheim′s sense, bounded in Pringsheim′s sense, regularly convergent, bounded and convergent respectively.

The notion of statistical convergence was first introduced by Fast [11] and Schoenberg [13] independently. Kumar [19] generalized this concept in probabilistic normed space. Khan and Khan ([20],[21]) introduced the I -convergence double sequence spaces by modulus function and sequence of modulii respectively. The notion of statistical convergent double sequence was introduced by Tripathy [4] and extended this concept in I-convergent double sequence. Tripathy and Sen [5] studied for double sequence space associated with multiplier sequences.

The notion of I-convergent is a generalization of the statistical convergence. I-convergence of real sequence was introduced at the initial stage by Kostyrko, Salat and Wilczynski [14]. Later on it was studied by Salat, Tripathy and Ziman [18] and many other researchers. Tripathy and Goswami [6] extended this concept in probabilistic normed space using triple difference sequences of real numbers.

At the initial stage the different types of notions of triple sequences was introduced and investigated by Sahiner, Gurdal and Duden [3]. Dutta, Esi and Tripathy [1] generalizes this concept by using Orlicz function. Savas and Patterson [10] introduced double sequence spaces defined by a modulus function. Sahiner and Tripathy [2] studied I-related properties in triple sequence spaces and showed some interesting results. Recently Tripathy and Goswami ([7],[8]) studied vector valued multiple sequences by using Orlicz function and multiple sequences in probabilistic normed spaces respectively. Debnath, Sharma and Das [16] and Debnath and Das [15] generalized these concepts by using the difference operator.

2. Definitions and Preliminaries

Definition 2.1. Let $X \neq \emptyset$. A class I of subsets of X is said to be an ideal in X provided:

(i) $\emptyset \in I$.

(ii) $A, B \in I$ implies $A \cup B \in I$.

(iii) $A \in I$, $B \subset A$ implies $B \in I$.

I is called a $non-trivial\ ideal$ if $X \notin I$.

Definition 2.2. Let $X \neq \emptyset$. A non empty class F of subsets of X is said to be a filter in X provided:

(i) $\emptyset \notin F$.

(ii) $A, B \in F$ implies $A \cap B \in F$.

(iii) $A \in F$, $A \subset B$ implies $B \in F$.

If I is a nontrivial ideal in X, $X \neq \emptyset$ then the class.

$F(I) = \{M \subset X : (\exists A \in I)(M = X \setminus A)\}$

is a filter on X, called the filter associated with I.

Definition 2.3. A nontrivial ideal I in X is called admissible if $\{x\} \in I$ for each $x \in X$.

Definition 2.4. A non-trivial ideal I is maximal if there cannot exist any non-trivial ideal $J \neq I$ containing I as a subset.

Definition 2.5. A triple sequence (x_{pqr}) is said to be convergent to L in Pringsheim's sense if for every $\varepsilon > 0$, there exists $n_0 \in N$ such that

$$|x_{pqr} - L| < \varepsilon \text{ , whenever } p \geq n_0,\ q \geq n_0,\ r \geq n_0,$$

and we write $\lim\limits_{p,q,r \to \infty} x_{pqr} = L.$

Note: A triple sequence convergent in Pringsheim's sense is not necessarily bounded [3]. This is clear from the following example.

Example 2.1. Consider the sequence (x_{pqr}) defined by

$$x_{pqr} = \begin{cases} p & for \quad all \quad p \in N, \quad q = 1 = r \\ \frac{1}{p+q+r} & otherwise \end{cases}$$

Then $x_{pqr} \to 0$ in Pringsheim′s sense but is unbounded.

Definition 2.6. A triple sequence (x_{pqr}) is said to be I-convergence to a number L if for every $\varepsilon > 0$, $\{(p,q,r) \in N \times N \times N : |x_{pqr} - L|) \geq \varepsilon\} \in I$. in this case we write $I - lim\ x_{pqr} = L$.

Definition 2.7. A triple sequence (x_{pqr}) is said to be I-null if $L = 0$. In this case we write $I - lim\ x_{pqr} = 0$.

Definition 2.8. A triple sequence (x_{pqr}) is said to be Cauchy sequence if for every $\varepsilon > 0$, there exists $n_0 \in N$ such that $|x_{pqr} - x_{lmn}| < \varepsilon$, whenever $p \geq l \geq n_0, q \geq m \geq n_0, r \geq n \geq n_0$.

Definition 2.9. A triple sequence (x_{pqr}) is said to be I-Cauchy if for every $\varepsilon > 0$, there exists $l = l_0$, $m = m_0$ and $n = n_0$ such that $\{(p,q,r) \in N \times N \times N : |x_{pqr} - x_{lmn}| \geq \varepsilon\} \in I$.

Definition 2.10. A triple sequence (x_{pqr}) is said to be I-bounded if there exists $M > 0$ such that $\{(p,q,r) \in N \times N \times N : |x_{pqr}| > M\} \in I$.

These classes are all linear spaces.

It is obvious that $c_0^3 \subset c^3$, $c^{3R} \subset c^{3B} \subset l_\infty^3$ and the inclusion are strict.

We state the following result without proof.

Theorem 2.1. *The spaces c_0^3, c^3, l_∞^3, c^{3R} and c^{3B} are complete normed linear spaces with the norm.*

$$\|x\| = \sup_{p,q,r} |x_{pqr}| < \infty$$

Definition 2.11. A triple sequence space E is said to be solid if $(\alpha_{pqr}x_{pqr}) \in E$ whenever $(x_{pqr}) \in E$ and for all sequences (α_{pqr}) of scalars with $|\alpha_{pqr}| \leq 1$, for all $p, q, r \in N$.

Definition 2.12. A triple sequence space E is said to be symmetric if $(x_{pqr}) \in E$ implies $(x_{\pi(p,q,r)}) \in E$, where π is a permutation of $N \times N \times N$.

Definition 2.13. A triple sequence space E is said to be monotone if it contains the canonical pre-images of all its step spaces.

Remark: A sequence space is solid implies that it is monotone.

Definition 2.14. Let $K = \{(i_p, j_q, k_r) : (p, q, r) \in N \times N \times N : i_1 < i_2 < i_3 < ...., j_1 < j_2 < j_3 < ...... \textit{ and } k_1 < k_2 < k_3 < ....\} \subseteq N \times N \times N$ and E is the triple sequence space. A K-stepspace of E is a sequence space $\lambda_K^E = \{(\alpha_{pqr}x_{pqr}) : (x_{pqr}) \in E\}$.

Definition 2.15. A triple sequence space E is said to be convergence free if $(y_{pqr}) \in E$, whenever $(x_{pqr}) \in E$ and $x_{pqr} = 0$ implies $y_{pqr} = 0$.

Definition 2.16. A triple sequence space E is said to be sequence algebra if $(x_{pqr} \star y_{pqr}) \in E$, whenever $(x_{pqr}) \in E$ and $y_{pqr} \in E$.

Lemma 2.1. Let $I \subset 2^N$ and $M \subseteq N$. If $M \notin I$, then $M \cap N \notin I$

Definition 2.17. A function $f : [0, \infty) \to [0, \infty)$ is called a modulus function if it satisfies the following four conditions [12]:

(1) $f(x) = 0$ if and only if $x = 0$

(2) $f(x + y) \leq f(x) + f(y)$ for all $x \geq 0$ and $y \geq 0$

(3) f is increasing.

(4) f is continuous from the right at 0.

Since $|f(x) - f(y)| \leq f(|x - y|)$, it follows from condition (4) that f is continuous on $[0, \infty)$. Furthermore, from condition (2) we have $f(nx) \leq nf(x)$, for all $n \in N$, and so

$f(x) = f(nx(\frac{1}{n})) \leq nf(\frac{x}{n})$, Hence $\frac{1}{n}f(x) \leq f(\frac{x}{n})$

for all $n \in N$. A modulus function may not be bounded. For example, $f(x) = x^p$, for $0 < p \leq 1$ is unbounded, but $f(x) = \frac{x}{1+x}$ is bounded.

Now we introduce the following triple sequence spaces, where $F = (f_{pqr})$ is the sequence of modulii :

$c^3_{0I}(F) = \{x \in w^3 : I - lim f_{pqr}(|x_{pqr}|) = 0\} \in I$

$c^3_I(F) = \{x \in w^3 : I - lim f_{pqr}(|x_{pqr} - L|) = 0$, for some $L\}\in$ I

$l^3_{\infty I}(F) = \{x \in w^3 : \underset{p,q,r \in N}{Sup} f_{pqr}(|x_{pqr}|) < \infty\}\in$ I

$m^3_I(F) = c^3_I(F) \cap l^3_{\infty I}(F)$

$m^3_{0I}(F) = c^3_{0I}(F) \cap l^3_{\infty I}(F)$

Where $c^3_{0I}(F)$, $c^3_I(F)$, $l^3_{\infty I}(F)$, $m^3_I(F)$ and $m^3_{0I}(F)$ are the triple sequence spaces I- null in Pringshiems sense , I- convergent in Pringshiems sense, I- bounded in Pringshiems sense, bounded I-convergence and bounded I- null in Pringshiems sense respectively.

The main aim of our study is to extend the concept of ideal convergence from double sequences to triple sequences defined by modulli function and establish some important results.

2. Main Results

Theorem 3.1. The triple sequence spaces $c_{0I}^3(F)$, $c_I^3(F)$, $l_{\infty I}^3(F)$, $m_I^3(F)$ and $m_{0I}^3(F)$ all are linear for the sequence of modulii $F = (f_{pqr})$.

Proof. We shall prove it for the sequence space $c_I^3(F)$, for the other spaces, it can be established similarly.

We assume that (x_{pqr}), $(y_{pqr})\in c_I^3(F)$ and α, β be two scalars such that $|\alpha| \leq 1$ and $|\beta| \leq 1$

Then

$I - lim\ f_{pqr}(|x_{pqr} - L_1|) = 0$, for some $L_1 \in C$

$I - lim\ f_{pqr}(|y_{pqr} - L_2|) = 0$, for some $L_2 \in C$

Now for a given $\varepsilon > 0$ we can write

$$A_1 = \{(p, q, r) \in N \times N \times N : f_{pqr}(|x_{pqr} - L_1|) > \frac{\varepsilon}{2}\} \in I \tag{2.1}$$

$$A_2 = \{(p, q, r) \in N \times N \times N : f_{pqr}(|y_{pqr} - L_2|) > \frac{\varepsilon}{2}\} \in I \tag{2.2}$$

Since f_{pqr} is a modulus function, so it is non-decreasing and convex, hence we get

$$f_{pqr}(|(\alpha x_{pqr} + \beta y_{pqr}) - (\alpha L_1 + \beta L_2)|) = f_{pqr}(|(\alpha x_{pqr} - \alpha L_1) + (\beta y_{pqr} - \beta L_2)|)$$

$$\leq f_{pqr}(|\alpha||x_{pqr} - L_1|) + f_{pqr}(|\beta||y_{pqr} - L_2|) = |\alpha| f_{pqr}(|x_{pqr} - L_1|) + |\beta| f_{pqr}(|y_{pqr} - L_2|)$$

$$\leq f_{pqr}(|x_{pqr} - L_1|) + f_{pqr}(|y_{pqr} - L_2|))$$

From (2.1) and (2.2) we can write

$$\{p,q,r \in N\times N\times N : f_{pqr}(|(\alpha x_{pqr}+\beta y_{pqr})-(\alpha L_1+\beta L_2)|) > \varepsilon\} \subset A_1\cup A_2$$

Thus $\alpha x_{pqr} + \beta y_{pqr} \in c_I^3(F)$

This completes the proof. □

Theorem 3.2. A triple sequence of real or complex numbers $x = (x_{pqr}) \in m_I^3(F)$ is I convergence iff for every $\varepsilon > 0$ there exist $I_\varepsilon, J_\varepsilon, K_\varepsilon \in N$ such that

$$\{(p,q,r) \in N\times N\times N : f_{pqr}(|x_{pqr} - x_{I_\varepsilon,J_\varepsilon,K_\varepsilon}|) \leq \varepsilon\} \in m_I^3(F)\}$$

Proof. Let $L = I{-}lim\ x$. Then we have

$A_\varepsilon = \{(p,q,r) \in N\times N\times N : f_{pqr}(|(x_{pqr} - L) \leq \frac{\varepsilon}{2}\} \in m_I^3(F)\}$ for all, $\varepsilon > 0$

Next fix $I_\varepsilon, J_\varepsilon, K_\varepsilon \in A_\varepsilon$ then we have

$|x_{pqr} - x_{I_\varepsilon,J_\varepsilon,K_\varepsilon}| \leq |x_{pqr} - L| + |L - x_{I_\varepsilon,J_\varepsilon,K_\varepsilon}| \leq \frac{\varepsilon}{2}+\frac{\varepsilon}{2} = \varepsilon$ for all, $p,q,r \in A_\varepsilon$

Thus $\{(p,q,r) \in N\times N\times N : f_{pqr}(|x_{pqr} - x_{I_\varepsilon,J_\varepsilon,K_\varepsilon}|) \leq \varepsilon\} \in m_I^3(f)\}$

Conversely, suppose that

$\{(p,q,r) \in N\times N\times N : f_{pqr}(|x_{pqr} - x_{I_\varepsilon,J_\varepsilon,K_\varepsilon}|) \leq \varepsilon\} \in m_I^3(F)\}$

we get $\{(p,q,r) \in N\times N\times N : f_{pqr}(|x_{pqr} - x_{I_\varepsilon,J_\varepsilon,K_\varepsilon}|) \leq \varepsilon\} \in m_I^3(F)\}$ for all, $\varepsilon > 0$

Then we can find the set

$B_\varepsilon = \{(p,q,r) \in N\times N\times N : x_{pqr} \in [x_{I_\varepsilon,J_\varepsilon,K_\varepsilon} - \varepsilon, x_{I_\varepsilon,J_\varepsilon,K_\varepsilon} + \varepsilon]\} \in m_I^3(F)$

given $\varepsilon > 0$.

Consider $N_\varepsilon = [x_{I_\varepsilon,J_\varepsilon,K_\varepsilon} - \varepsilon, x_{I_\varepsilon,J_\varepsilon,K_\varepsilon} + \varepsilon]$

Now we have $B_\varepsilon \in m_I^3(F)$ as well as $B_{\frac{\varepsilon}{2}} \in m_I^3(F)$

Hence $B_\varepsilon \cap B_{\frac{\varepsilon}{2}} \in m_I^3(F)$ which implies $M_\varepsilon \cap M_{\frac{\varepsilon}{2}} \neq \emptyset$

Then $\{(p,q,r) \in N \times N \times N : x_{pqr} \in N\} \in m_I^3(F)$

which implies $diamM \leq diamM_\varepsilon$

where the diam of M denotes the length of interval N.

In this way by principal of induction we found the sequence of closed intervals

$M_\varepsilon = I_0 \supseteq I_1 \supseteq I_2 \supseteq .... \supseteq I_s \supseteq ....$

With the help of the property that $diamI_s \leq \frac{1}{2} diamI_{s-1}$, for $(s = 1,2,3,4, ...)$ and $\{(p,q,r) \in N \times N \times N : x_{pqr} \in I_{pqr}\} \in m_I^3(F)$, for $(p,q,r = 1,2,3,4, ...)$

Then there exist a $\xi \in \cap I_s$ where $s \in N$ such that $\xi = I - lim\ x$

So that $f_{pqr}(\xi) = I - lim\ f_{pqr}(x)$ therefore $L = I - lim\ f_{pqr}(x)$

Hence the proof is complete. □

Theorem 3.3. The inclusions $c_{0I}^3(F) \subset c_I^3(F) \subset l_{\infty I}^3(F)$ are strict, where F is the function of modulii.

Proof. We consider $x_{pqr} \in c_I^3(F)$. Then there exist $L \in C$ such that

$I - lim\ f_{pqr}(|x_{pqr} - L|) = 0$, we get $f_{pqr}(|x_{pqr}|) \leq f_{pqr}(|x_{pqr} - L|) + f_{pqr}(|L|)$

On taking supremum over p , q and r on both sides gives

$(x_{pqr}) \in l_{\infty I}^3(F)$

The inclusion $c^3_{0I}(F) \subset c^3_I(F)$ is obvious

Hence the inclusion $c^3_{0I}(F) \subset c^3_I(F) \subset l^3_{\infty I}(F)$ are strict. □

Theorem 3.4. The triple sequence spaces $c^3_{0I}(F)$ and $m^3_{0I}(F)$ are solid.

Proof. We shall prove the result for $c^3_{0I}(F)$ and the result for $m^3_{0I}(F)$ can be established similarly .

Consider $x_{pqr} \in c^3_{0I}(F)$ and then $I - \underset{pqr}{lim}\ f_{pqr}(|x_{pqr}|) = 0$

Now we consider a sequence of scalar (α_{pqr}) such that $|\alpha_{pqr}| \leq 1$, for all $p, q, r \in N$

Then we have,

$I - \underset{pqr}{lim}\ f_{pqr}(|\alpha_{pqr} x_{pqr}|) \leq I - \underset{pqr}{lim}\ f_{pqr}(|\alpha_{pqr}||x_{pqr}|)$

$= |\alpha_{pqr}| I - \underset{pqr}{lim}\ f_{pqr}(|x_{pqr}|) = 0$

Hence $I - \underset{pqr}{lim}\ f_{pqr}(|\alpha_{pqr} x_{pqr}|) = 0$ for all $p, q, r \in N$

This implies $\alpha_{pqr} x_{pqr} \in c^3_{0I}(F)$

Therefor the space $c^3_{0I}(F)$ is solid.

Hence the proof. □

Theorem 3.5. The triple sequence spaces $c^3_I(F)$ and $m^3_I(F)$ are not monotonic in general.

Proof. We take a counter example to prove the result for $c^3_I(F)$. The other part can be proved similarly.

Let $I = I_\delta$ and $f(x) = x$, for all $x \in [0, \infty)$

Consider the K-step space $X_K(F)$ of X defined as follows:

Now we assume $x_{pqr} \in X$ and $y_{pqr} \in X_K$ such that

Sequence (x_{pqr}) is defined by $x_{pqr} = 1$, for all $p, q, r \in N$

and sequence (y_{pqr}) is defined by

$$(y_{pqr}) = \begin{cases} (x_{pqr}), & when \quad p, \quad q, \quad r \quad are \quad even \\ 0, & otherwise \end{cases}$$

Then $(x_{pqr}) \in c_I^3(F)$ but its K-step space pre-image does not belong to $c_I^3(F)$.

Therefore the sequence space $c_I^3(F)$ is not monotonic.

This complete the proof. □

Theorem 3.6. The triple sequence spaces $c_{0I}^3(F)$, $c_I^3(F)$, $l_{\infty I}^3(F)$, $m_I^3(F)$ and $m_{0I}^3(F)$ are sequence algebras.

Proof. To prove it we consider the sequence space $c_{0I}^3(F)$, for the rest of the cases the proof will follow similarly.

Let $(x_{pqr}), (y_{pqr}) \in c_{0I}^3(F)$

Then we have $I - lim\ f_{pqr}(|x_{pqr}|) = 0$, and $I - lim\ f_{pqr}(|y_{pqr}|) = 0$,

Now we obtain $I - lim\ f_{pqr}(|x_{pqr}.y_{pqr}|) = 0$,

It implies that $(x_{pqr}.y_{pqr}) \in c_{0I}^3(F)$ is a sequence algebra

Hence the proof. □

Theorem 3.7. The sequence spaces $c_{0I}^3(F)$, $c_I^3(F)$ and $l_{\infty I}^3(F)$ are not convergence free in general.

Proof. To prove it we consider a counter example for the space $c_I^3(F)$.

Let $I = I_f$ and $f(x) = x$, for all $x \in [0, \infty)$

Now we consider two triple sequence (x_{pqr}) and (y_{pqr}) such that

$(x_{pqr}) = \frac{1}{p+2q+3r}$, for all $p, q, r \in N$

and

$(y_{pqr}) = p + 2q + 3r$, for all $p, q, r \in N$

It can be easily verified that $(x_{pqr}) \in c_I^3(F)$ and $(y_{pqr}) \notin c_I^3(F)$

Hence the space $c_I^3(F)$ is not convergence free.

Similarly the others. $\square$

Theorem 3.8. If I is neither maximal nor $I \neq I_f$ then the triple sequence spaces $c_{0I}^3(F)$ and $c_I^3(F)$ are not symmetric in general.

Proof. Consider $A \in I$ be infinite and $f(x) = x$, for all $x \in [0, \infty)$.

If $(x_{pqr}) = \begin{cases} 2, & for \quad all \quad p, \quad q, \quad r \quad \in A \\ 0, & otherwise \end{cases}$

Then by Lemma 2.1 we can have $(x_{pqr}) \in c_{0I}^3(F) \subset c_I^3(F)$.

Now we consider $K \subset N$ such that $K \notin I$ and $N - K \notin I$.

Let $\chi : K \rightarrow A$ and $\psi : N - K \rightarrow N - A$ be bijections, then the map $\pi : N \rightarrow N$ defined by

$$\pi(pqr) = \begin{cases} \chi(p, q, r), & for \quad all \quad p, \quad q, \quad r \quad \in K \\ \psi(p, q, r), & otherwise \end{cases}$$

is a permutation on N, but $\{x_{(\pi(p,q,r))}\} \notin c_{0I}^3(F)$ and $\{x_{(\pi(p,q,r))}\} \notin c_I^3(F)$.

Hence the sequence spaces $c_{0I}^3(F)$ and $c_I^3(F)$ are not symmetric.

This complete the proof. $\square$

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Bimal Chandra Das
Department of Mathematics
Govt. Degree College,
Kamalpur - 799285
Tripura University
Dhalai, Tripura,
INDIA
e-mail : bcdas3744@gmail.com